Chapter 10

2. The standard deviation

4. $SS_1 = 80$

$SS_2 = 48$

$n = 17$

a) $S_1 = \sqrt{\frac{SS_1}{n-1}} = \sqrt{\frac{80}{16}} = 2.236$ $\quad S_1^2 = 5$

$S_2 = \sqrt{\frac{SS_2}{n-1}} = \sqrt{\frac{48}{16}} = 1.732$ $\quad S_2^2 = 3$

$$S^2p = \frac{(n-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{16(5)+3(16)}{32} = 4$$

b) $n_1 = 17$

$n_2 = 5$

$S_1^2 = \frac{SS1}{n_1-1} = \frac{80}{16} = 5$

$S_2^2 = \frac{SS_2}{n_2-1} = \frac{48}{4} = 12$

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{5(16)+4(12)}{20} = 6.4$$

6. $n_1 = 5$, $SS_1 = 60$

$n_2 = 9$, $SS_2 = 84$

a) $S_1^2 = \frac{SS_1}{n_1-1} = \frac{60}{4} = 15$

$S_2^2 = \frac{SS_2}{n_2-1} = \frac{84}{9-1} = 10.5$

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{4 \times 15 + 9(10.5)}{12} = 12.875$$

b) Estimated standard error $= \sqrt{\frac{Sp^2}{n_1} + \frac{Sp^2}{n_2}} = \sqrt{\frac{12.875}{15} + \frac{12.875}{9}} = 2.001$

c) $d_f = n_1 + n_2 - 2 = 5 + 9 - 2 = 12$

$$t = \frac{M_1 - M_2}{S_p^2} = \frac{7}{2} = 3.5$$

$t_{12, 0.05} = \pm 2.1788$

Yes, we reject the null hypothesis.

8. $M = 786$, $S = 85$ $n = 100$

$M = 772$ $S = 91$ $n = 100$

$$S_p^2 = \frac{(n-1)S_1^2 + (n-1)S_2^2}{n_1 + n_2 - 2} = \frac{98(85^2) + 98(91^2)}{198} = 7674.69$$

$$SE = \sqrt{\frac{Sp^2}{n_1} + \frac{Sp^2}{n_1}} = \sqrt{\frac{7674.69}{100} + \frac{7674.69}{100}} = 12.3893$$

$$t_{stat} = \frac{M_1 - M_2}{SE} = \frac{786 - 772}{12.3893} = 1.13$$

$d_f = n_1 + n_2 - 2 = 200 - 2 = 198$

$t_{198, 0.05} = \pm 1.9840$

The null hypothesis is not rejected because our $t_{n-1, \alpha}$ is more than t_{stat} and therefore t_{stat} is in the acceptance region.

b) r^2

$$d = \frac{M_1 - M_2}{S} \qquad S = \frac{1}{2}(S_1 + S_2) = \frac{85 + 91}{2} = 88$$

$$d = \frac{786 - 772}{88} = 0.1591$$

$$r^2 = \left(\frac{d}{\sqrt{d^2 + 4}}\right)^2 = \left(\frac{0.1591}{\sqrt{0.1591^2 + 4}}\right)^2 = 0.006288$$

10. a) $n = 27$ $\quad n = 27$

$M_1 = 7.41$ $\quad M = 4.78$

$SS_1 = 749.5$ $\quad SS_2 = 830$

$$S_1 = \frac{SS_1}{n-1} = \frac{749.5}{26} = 28.83$$

$$S_2 = \frac{SS_2}{n-1} = \frac{830}{26} = 31.92$$

$$Sp^2 = \frac{(n_1-1)S_1^2 + (n-1)S_2^2}{n_1+n_2-2} = \frac{26(28.83^2) + 26(31.92^2)}{52} = 925.03$$

$$SE = \sqrt{\frac{Sp^2}{n_1} + \frac{Sp^2}{n_2}} = \sqrt{\frac{925.03}{27} + \frac{925.03}{27}} = 34.26$$

$$t_{stat} = \frac{M_1 - M_2}{SE} = \frac{749.5 + 830}{34.26} = + 2.3496$$

$$df = n_1 + n_2 - 2 = 27 + 27 - 2 = 52$$

$$t_{52, 0.a} = \cancel{\pm 2.0066} + 1.6746$$

The null hypothesis is rejected and therefore high creativity people are more likely to cheat than people of low creativity.

b) $d = \frac{M_1 - M_2}{S}$ $\quad S = \frac{1}{2}(S_1 + S_2) = \frac{31.92 + 28.83}{2}$

$d = \frac{7.41 - 4.78}{30.375}$ $\quad S = 30.375$

$d = 0.08658$

c) Because t statistics is larger ~~lower~~ than the critical value, we reject the null hypothesis and conclude that high creativity people are more likely to cheat than people of low creativity. The effect size is small.

12. $n = 16$ $\quad n = 16$

$M_1 = 5.75$ $\quad M_2 = 5.00$

$SS = 6.5$ $\quad SS = 10.0$

$S_1 = \frac{SS_1}{n-1} = \frac{6.5}{15} = 0.4333$ $\qquad$ $S_2 = \frac{SS_2}{n-1} = \frac{10.0}{15} = 0.667$

$S_p^2 = \frac{(n-1)S_1^2 + (n-1)S_2^2}{n_1+n_2-2} = \frac{(15)\,0.433^2 + 15(0.667^2)}{30} = 0.3162$

$SE = \sqrt{\frac{Sp^2}{n_1} + \frac{Sp^2}{n_2}} = \sqrt{\frac{0.3162}{16} + \frac{0.3162}{16}} = 0.1988$

$t_{stat} = \frac{M_1 - M_2}{SE} = \frac{5.75 - 5.00}{0.1988} = 3.7726$

$df = 16 + 16 - 2 = 30$

$t_{30,\,0.01} = 2.4573$

The null hypothesis is not rejected and therefore we conclude that unexpected positive outcome did not increase the amount of money participants were willing to gamble.

14. $n = 68$

$M_1 = \frac{1}{8}(37 + 35 + 34 + 36 + 40 + 34 + 33 + 39) = 36$

$M_2 = \frac{1}{8}(34 + 31 + 36 + 30 + 34 + 33 + 37 + 29) = 33$

	$X - \bar{X}$	$(X-\bar{X})^2$
37	1	1
35	-1	1
34	-2	4
36	0	0
40	4	16
34	-2	4
33	-3	9
39	3	9
		$\Sigma = 44$

	$X - \bar{X}$	$(X-\bar{X})^2$
34	1	1
31	-2	4
36	3	9
30	-3	9
34	1	1
33	0	0
37	4	16
29	-4	16
		$\Sigma = 56$

$S_1^2 = \frac{\Sigma(X-\bar{X})^2}{n_1}$

$S_1^2 = \frac{44}{68} = 0.6471$

$S_2^2 = \frac{\Sigma(X-\bar{X})^2}{n_2} = \frac{56}{68} = 0.8235$

$$Sp^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{(67)(0.6471^2) + 67(0.8235^2)}{134} = \cancel{0.5484}\ 0.7353$$

$$SE = \sqrt{\frac{Sp^2}{n_1} + \frac{Sp^2}{n_2}} = \sqrt{\frac{\cancel{0.5484}\ 0.7353}{68} + \frac{\cancel{0.5484}\ 0.7353}{68}} = 0.1470$$

$$t_{stat} = \frac{|M_1 - M_2|}{SE}$$

$$S = \frac{1}{2}(S_1 + S_2) = \frac{1}{2}(0.6471 + 0.8235)$$

$$S = 0.7353$$

$$= \frac{36-33}{\cancel{0.7353}\ 0.1470} = \cancel{4.07996}\ 20.40$$

$$t_{df,\alpha} = t_{134,\,0.05} = \pm 1.9778$$

The null hypothesis is rejected

b). 95% confidence interval

$$(M_1 - M_2) \pm t \cdot s/\sqrt{n}$$

$$3 \pm 1.9778 \cdot \frac{0.7353}{\sqrt{69}} = 3 \pm 0.1751 = (2.8249, 3.1751)$$

c) The test statistic is greater than the critical value therefore it lies in the rejection region and hence we reject the null hypothesis and conclude that the participants who received donuts on paper plate wasted more food than participants who were served donuts on platters.

16. $t(20) = 2.09$

a) $df = 20 = n_1 + n_2 - 2 \Rightarrow n_1 + n_2 = df + 2 = \underline{22}$

b) $\alpha = 0.05$

$$t_{20,\,0.05} = \pm 2.08596 = \pm 2.0860$$

The null hypothesis is not rejected, therefore there is significant difference between the two treatments.

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c) $t_{20, 0.01} = \pm 2.8453$

t-static is less than the critical value and therefore the null hypothesis is not rejected. Therefore no significant difference.

18. This is an assumption of the independent samples t-test and ANOVA stating all groups have the same variance. It tells us whether the independent variables we have chosen are statistically significant.

20. a) $n_1 = 6 \quad SS_1 = 500 \quad , n_2 = 12 \quad , SS_2 = 524$

$$S_1 = \frac{SS_1}{n_1 - 1} = \frac{500}{5} = 100 \qquad S_2 = \frac{SS_2}{n_2 - 1} = \frac{524}{11} = 47.64$$

$$Sp^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{5(100^2) + 11(47.64)^2}{16} = 4685.3291$$

$$SE = \sqrt{\frac{Sp^2}{n_1} + \frac{Sp^2}{n_2}}$$

$$SE = \sqrt{\frac{4685.33}{6} + \frac{4685.33}{12}} = 34.224$$

b) $n_1 = 6 \quad , SS_1 = 600 \quad \& \quad n_2 = 12 \quad SS_2 = 696$

$$S_1 = \frac{SS_1}{n_1 - 1} = \frac{600}{5} = 120 \qquad S_2 = \frac{SS_2}{n_2 - 1} = \frac{696}{11} = 63.27$$

$$Sp^2 = \frac{(5)(120^2) + 11(63.27^2)}{16} = 7252.13$$

$$SE = \sqrt{\frac{Sp^2}{n_1} + \frac{Sp^2}{n_2}} = \sqrt{\frac{7252.13}{6} + \frac{7252.13}{12}} = 42.58$$

c) Higher variability increases the standard error for sample mean difference.

22. a) $n_1 = 7 \quad S_1^2 = 142$

$n_2 = 7 \quad S_2^2 = 110$

$$Sp^2 = \frac{(n-1)S_1^2 + (n-1)S_2^2}{n_1+n_2-2} = \frac{6(142) + 6(110)}{12} = 126$$

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(b) $n_1 = 28$, $s_1^2 = 142$

$n_2 = 28$, $s_2^2 = 110$

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{27(142) + 27(110)}{54} = 126$$

(c) The sample sizes do not change the pooled variance.

24. $n_1 = 15$ $n_2 = 15$

$M = 40.8$ $M = 34.9$

$SS = 510$ $SS_2 = 414$

$$S_1 = \frac{SS_1}{n-1} = \frac{510}{14} = 36.43 \qquad S_2 = \frac{SS_2}{n-1} = \frac{414}{14} = 29.57$$

$$S_p^2 = \frac{(n-1)S_1^2 + (n-1)S_2^2}{n_1+n_2-2} = \frac{14(36.43^2) + 14(29.57^2)}{28} = 1100.76$$

$$SE = \sqrt{\frac{1100.76}{15} + \frac{1100.76}{15}} = 12.115$$

$$S = \frac{1}{2}(S_1 + S_2) = \frac{31.43 + 29.57}{2} = 33$$

$$t\,stat = \frac{|M_1 - M_2|}{SE} = \frac{40.8 - 34.9}{\cancel{33}\ 12.115} = \cancel{0.17488}\ 0.48699$$

$$t_{df,\alpha} = t_{28,\,0.05} = \pm 2.0484$$

The null hypothesis is not rejected, therefore no significant diff.

$$d = \frac{M_1 - M_2}{S} = 0.1788$$

$$r^2 = \left(\frac{d}{\sqrt{d^2+4}}\right)^2 = \left(\frac{0.1788}{\sqrt{0.1788^2+4}}\right)^2 = 0.0079$$

b) $SS_1 = 1020$ $SS_2 = 828$

$$S_1 = \frac{SS_1}{n-1} = \frac{1020}{14} = 72.86$$

$$S_2 = \frac{SS_2}{n-1} = \frac{828}{14} = 59.14$$

$$S_p^2 = \frac{(14)(72.86^2) + 14 \times 59.14^2}{28} = 4403.06$$

$$SE = \sqrt{\frac{4403.06}{14} + \frac{4403.06}{14}} = 25.08$$

$$t_{stat} = \frac{|m_1 - m_2|}{SE} = \frac{40.8 - 34.9}{25.08} = 0.23524$$

$$d = \frac{m_1 - m_2}{s} = \frac{40.8 - 34.9}{66} = 0.08939$$

$$r^2 = \left[\frac{d}{\sqrt{d^2+4}}\right]^2 = \left[\frac{0.08939}{\sqrt{0.08939^2+4}}\right]^2 = 0.00199$$

$$t_{28,0.05} = \pm 2.0484$$

c) Variability has no effect on hypothesis outcome but it decreases the value of measure of effect when increased.